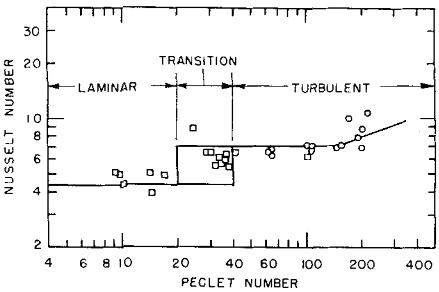

Fig. 3. Plot of Nusselt number vs. Reynolds number.

should have a minimal effect upon the the heat transfer coefficient.

The accuracy of the data obtained from this experiment was checked by means of an error analysis. The experimental uncertainties in the values for Nusselt number were found to range from ± 6 to ± 16% for the square data points in Figures 2 and 3, and uncertainties ranging from ± 10 to ± 22% were found for the round data points. The uncertainty in the Reynolds numbers was ± 5% and the uncertainty in the Prandtl numbers was ± 2%. The possible errors in the data were determined by using the statistical method. This method consists of taking the square root of the sum of the squares of the individual experimental uncertainties to determine the possible value of error.

The accuracy of the Nusselt number calculated from experimental data is largely dependent upon accurate determination of the difference between the inside wall temperature and the bulk fluid temperature. In the determination of the possible error in calculating the Nusselt number, we considered the following possible sources of error: heat flux, thermal conductivity of SS tubing, heater inlet temperature, heater outlet temperature, and outside wall temperature. In determining the accuracy of the Reynolds numbers, we considered the accuracies with which the flow rate could be experimentally determined and of the published thermophysical property data (*8*). The accuracy of the thermophysical property data was also considered in determination of the Prandtl numbers.

The data points shown in Figures 2 and 3 represent an average of three to eight individual data points at which the flow rates and bulk fluid temperatures were identical; thus the Reynolds numbers and Prandtl numbers were identical. The accuracy of these individual data points was within the experimental accuracy limits determined by the error analysis.

Some of the earlier results which showed the Nusselt numbers to be somewhat lower than predicted by Equations (1) and (2) may be explained by nonwetting of the heat transfer surface by the liquid metal. Recent data (*3*, *9*) have extended the temperature range of these experiments with the alkali metals, thus assuring complete wetting of the heat transfer surface. These data (*3*, *9*) are in agreement with Equations (1) and (2).

The data from this experiment support the equations presented by Lyon (*2*) and Dwyer (*1*) for estimating heat transfer rates to liquid metals during vertical upflow.

ACKNOWLEDGMENT

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NOTATION

D_o = O.D. of heated tube, ft.
D_i = I.D. of heated tube, ft.
K = thermal conductivity of tube, B.t.u./(hr.)(ft.)(°F.)
L = heated length, ft.
N_{Nu} = Nusselt number, dimensionless
N_{Pe} = Peclet number, dimensionless
N_{Pr} = Prandtl number, dimensionless
q = heat conducted through the tube, B.t.u./hr.
N_{Re} = Reynolds number, dimensionless
T_o = local outside tube wall temperature, °F.
T_i = local inside tube wall temperature, °F.

Greek Letters

ϵ_M = eddy diffusivity for momentum transfer, sq.ft./hr.
ν = kinematic viscosity, sq.ft./hr.

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A Criterion for Short and Long Wetted-Wall Columns

JOHN A. TALLMADGE

Imperial College of Science and Technology, London, England

A simple and convenient length criterion for practical purposes is whether the Fourier number is more or less than 0.1.

Consider the case of fully developed laminar flow of a liquid B down the walls of a cylinder with a radius sufficiently large so that the effects of curvature are negligible. Impose on this system mass transfer of component *A* from a gas such that the concentration of *A* at the liquid-gas interface is constant with length. Mass transfer in this liquid phase has been shown by several authors to be

$$U_s\left[1-\left(\frac{x}{h}\right)^2\right]\frac{\partial C}{\partial z} = D_{AB}\frac{\partial^2 C}{\partial x^2} \quad (1)$$

$$C = C_o \quad \text{at} \quad z = 0 \quad (2)$$

$$C = C_s \quad \text{at} \quad x = 0 \quad (3)$$

$$\partial C/\partial x = 0 \quad \text{at} \quad x = h \quad (4)$$

These equations are written with the coordinate origin at the top of the liquid-gas interface.

Equation (1) is precise only where the molecular diffusion of *A* in the *z* direction is negligible compared to transport of *A* by fluid motion in the *z* direction, and where conditions are such that the difference between the flux of *A* in Lagrangian and Eulerian forms is negligible.

Equation (1) has been solved by Pigford (*3*), who used boundary con-

INFORMATION RETRIEVAL

(*Continued from page 1152*)

Partial oxidation of o-xylene in melts containing vanadium pentoxide, Satterfield, Charles N., and Jordan Loftus, **A.I.Ch.E. Journal, 11,** No. 6, p. 1103 (November, 1965.

Key Words: A. Oxidation-8, o-Xylene-1, Nitrogen-5, Melts-10, Vanadium Pentoxide-10, Potassium Sulfate-10, o-Tolualdehyde-2, Benzene-2, Phthalide-2, Bubbles-1, Catalyst-10, Melt Reactor-10, Bubble Reactor-10.

Abstract: The purpose of this work was to explore a different contacting method, that of passing the reactant gas in the form of bubbles up through a vessel containing a melt which acts as an oxidizing agent or as a catalyst.

O-xylene vapor in nitrogen was partially oxidized by bubbling it through a melt of vanadium pentoxide-potassium sulfate eutectic. The principal product was o-tolualdehyde; some benzene and traces of phthalide were formed but no organic acids.

The determination of eddy mass diffusivities for the air-water system in a wetted-wall column, Bunch, David W., and Mailand R. Strunk, **A.I.Ch.E. Journal, 11,** No. 6, p. 1108 (November, 1965).

Key Words: A. Reynolds Number-6, Position-6, Radius-9, Eigenvalue-7, Eddy Mass Diffusivity-7, 8, Air-9, Column-9, Concentration Profiles-7, Mathematics-10, Digital Computer-10, Mass Transfer-8.

Abstract: The purpose of this study was to determine the eddy mass diffusivity values as a function of radial position at various Reynolds numbers ranging from 3,200 to 54,300 used. Concentration profiles were obtained with a small lithium chloride probe. A mathematical procedure was formulated so that the eddy values could be evaluated from experimental observations with a digital computer.

A study of slip ratios for the flow of steam-water mixtures at high void fractions, Vance, William H., and R. W. Moulton, **A.I.Ch.E. Journal, 11,** No. 6, p. 1114 (November, 1965).

Key Words: Slip Ratios-8, 7, Fluid Flow-8, Two-Phase-0, Void Fraction-8, Steam-5, Water-5, Flow-8, Tube-10, Adiabatic-0, Quality-6, Pressure-6, Velocity-6, Pressure Gradient-6, Computer-10, Theoretical-0, Calculations-8.

Abstract: A technique employing the impulse plate principle was developed whereby the ratio of the average vapor-to-liquid velocities (slip ratio) for flowing two-phase mixtures could be measured accurately at high vapor volume fractions. Data were collected for steam-water mixtures flowing adiabatically in a horizontal tube. The experimental slip ratio decreased with increasing quality and pressure and increased with increasing mass velocity and pressure gradient.

Heat transfer to evaporating refrigerants in two-phase flow, Lavin, J. Gerard, and Edwin A. Young, **A.I.Ch.E. Journal, 11,** No. 6, p. 1124 (November, 1965).

Key Words: A. Heat Transfer-8, Heat Transfer Coefficient-8, Refrigerants-9, Two-Phase Flow-9, Freon-12 -9, Freon-22 -9, Nucleate Boiling-8, Annular Flow-8, Mist Flow-8, Evaporation-8, Flow Regimes-8, Tubes-9, Finned Tubes-9.

Abstract: Flow regimes occurring during evaporation of Freon-12 and Freon 22 inside fine tubes were investigated. Two flow models are proposed, one each for vertical and horizontal tube orientation. The limits of each flow regime were determined and heat transfer correlations obtained. The most significant flow regimes were found to be: nucleate boiling, annular flow, and mist flow. A correlation for the transition between annular flow and mist flow was obtained.

ditions of Equation (2), (3), and (4) to obtain $C = C(x,z)$. The result, expressed by integration in terms of the average bulk concentration C_B at length $z = L$, is

$$1 - R = E(N_{Fo}) \qquad (5)$$

where

$$R = (C_B - C_o)/(C_s - C_o) \qquad (6)$$

and

$$E(N_{Fo}) = 0.78\, e^{-5.1 N_{Fo}} + 0.10\, \mathbf{e}^{-39.3 N_{Fo}} + 0.04\, e^{-105.6 N_{Fo}} + \ldots . \qquad (7)$$

Here R is the fraction absorbed and the Fourier number is the dimensionless contact time.

It is convenient to consider N_{Fo} as the dimensionless length for mass transport, as shown in Equation (8):

$$N_{Fo} = \left(\frac{D_{AB}}{h^2}\right)\theta = \left(\frac{D_{AB}}{h^2 U_s}\right) L \qquad (8)$$

Equation (5) is an *exact* solution for Equations (1) to (4) but is in a form which is convergent for columns which are long in the Fourier number sense.

Special solutions of Equations (1) to (4) have been obtained by considering tubes short enough so that concentration gradients do not arise at $x = h$; thus Equation (4) is replaced by a penetration theory type of boundary condition:

$$\partial C/\partial x = 0 \quad \text{at} \quad x \to \infty \qquad (9)$$

An even more restrictive solution has been obtained for tube so short that transfer in the region of velocity gradients is neglected, that is where Equation (1) is replaced by

$$U_s \frac{\partial C}{\partial x} = D_{AB} \frac{\partial^2 C}{\partial x^2} \qquad (10)$$

The results obtained from solutions of these special cases by using the boundary conditions of Equations (2), (3), and (9) are well known. Used with Equation (1), a *medium* length tube solution is obtained, which may be indicated as

$$R = M(N_{Fo}) \qquad (11)$$

Used with Equation (10), the following, more restricted result is obtainable for a *short* tube (1):

$$R = S(N_{Fo}) \qquad (12)$$

where

$$S(N_{Fo}) = (9\, N_{Fo}/\pi)^{1/2} = 1.69\, N_{Fo}^{\ 1/2} \qquad (13)$$

The question to be answered is: For what lengths, in the N_{Fo} sense, are the various solutions valid? The answer is surprisingly simple. It may be developed as follows.

First the length is determined for which the exact function of Equation

(7) can be replaced, within 1% in $1 - R$, by the first term in the series; thus

$$(1.01)\, P(N_{Fo}) = E(N_{Fo}) \qquad (14)$$

where

$$P(N_{Fo}) = 0.78\, e^{-5.1 N_{Fo}} \qquad (15)$$

Since the value of N_{Fo} which satisfies Equation (14) is 0.08, the *approximate* function, Equation (15), is practically equivalent to the exact function, Equation (7), for all $N_{Fo} \geqq 0.08$. For these values of N_{Fo}, $R \geqq 0.48$.

Second, the length is determined for which the short tube function, Equation (13), is equal, within 1% in $1 - R$, to the approximate function, Equation (15), or

$$1 - S(N_{Fo}) = (1.01)\, P(N_{Fo}) \qquad (16)$$

Since the value of N_{Fo} which satisfies Equation (16) is about 0.1, this short tube solution is practically equivalent to the exact solution of Equation (5) for all $N_{Fo} \leqq 0.1$. For these values of N_{Fo}, $R \leqq 0.53$.

Based on these two results, a simple and convenient criterion for column length is whether the Fourier number is more or less than 0.1. If more, the tube is long; if less, the tube is short. It seems fortuitous that there is no need for equations describing an intermediate length region. As a result, however, only simple functions are necessary to predict R and $1 - R$ within 1% for all tube lengths. Therefore, for long tubes ($N_{Fo} \geqq 0.1$)

$$1 - R = 0.78\, e^{-5.1 N_{Fo}} \qquad (17)$$

For short tubes ($N_{Fo} \leqq 0.1$)

$$R = 1.69\, N_{Fo}^{1/2} \qquad (12)$$

The values of the normalized fraction absorbed R are greater and less than 0.5 for Equations (17) and (12), respectively; this midpoint value of 0.5 also seems somewhat fortuitous.

These two equations provide more precise values and a better analytical description of the functions than small-scale graphs do (3).

Because Equations (17) and (12) describe R so precisely over all applicable lengths and conditions, it follows that there is no need to use the exact solution, Equation (5). Furthermore, other approximate solutions become, for practical purposes, only of historical or academic interest. Such special equations which are replaced by Equations (17) and (12) include the medium-length tube solution indicated by Equation (11) and the solution recommended by Sherwood and Pigford (*4*), Equation (18)

$$1 - R = \frac{8}{\pi^2}\left(e^{-\pi N_{Fo}/4} + \frac{1}{9} e^{-9\pi^2 N_{Fo}/4} + \ldots \right) \qquad (18)$$

Equation (18) was obtained by solving the approximate equation (10) with the boundary conditions of Equations (2) to (4).

In terms of length L, therefore, whether a tube is considered long or short depends on the following criterion, which is based on $N_{Fo} = 0.1$ and Equation (8). For long tubes

$$L \geqq 0.1\, U_s h^2 / D_{AB} \qquad (19)$$

From fluid dynamics it is known (2) that the surface velocity and film thickness are parameters related by

$$U_s = \rho g h^2 / 2\mu \qquad (20)$$

There is a temptation to use Equation (20) together with (19) to obtain a criterion in terms of U_s or h, as well as liquid properties. However, neither form would be particularly helpful for design purposes, since it is frequently more useful to specify flow rate rather than fluid velocity or thickness. Mass flow rate may be introduced by the following relationship (2):

$$h^3 = 3\mu\Gamma / \rho^2 g \qquad (21)$$

Combining Equations (19), (20), and (21), one obtains

$$L \geqq \frac{0.22}{D_{AB}} \left(\frac{\mu\Gamma^4}{\rho^5 g} \right)^{1/3} \qquad (22)$$

Equation (22) is the criterion for tubes to be long in the $N_{Fo} > 0.1$ sense and is expressed as a function of mass flow rate and fluid properties only.

The usefulness of Equation (22) may be illustrated by proving that Vivian and Peaceman's work (5) was conducted under short tube conditions. To do this one must show that the longest length actually used (4.25 cm.) was less than the long tube length criterion of Equation (22) for the smallest flow rate [0.12 g./(sec.)(cm.)] and largest diffusivity (1.96 × 10^{-5} sq. cm./sec.). The fluid viscosity and density were approximately 0.01 poise and 1.0 g./cc., respectively. With these parameters L from Equation (22) = 14 cm.; thus all experiments were done in the short tube region for which Equation (12) is valid. In other words, the maximum Fourier number used by Vivian and Peaceman (0.03) was slightly below the maximum for the short tube solution.

Another example illustrating the usefulness of Equation (22) is the determination of the maximum length allowable for laminar short tube transport. Substitution of critical Reynolds number for wave-free flow causes Equation (22) to become

$$L \geqq \frac{0.22}{D_{AB}} \left(\frac{\mu^5 N_{Re_c}{}^4}{\rho^5 g} \right)^{1/3} \qquad (23)$$

For $N_{Re_c} \leqq 4$ (2) then, long tubes are described by Equation (24):

$$L \geqq \frac{0.14}{D_{AB}} \left(\frac{\mu}{\rho} \right)^{5/3} \qquad (24)$$

Thus, for water having a temperature such that the viscosity is 0.01 poise and the density 1 g./cc. and a substance having a diffusivity of 10^{-5} sq. cm./sec., the length criterion for the system is 6.5 cm.

NOTATION

A = substance A
B = substance B
C = concentration of A in liquid
D_{AB} = molecular diffusivity of AB, sq. cm./sec.
E = function of N_{Fo}, exact solution, Equation (7)
g = acceleration of gravity, 980 cm./sec.^{2}
h = thickness of liquid film, cm.
L = column or tube length, cm.
M = function of N_{Fo}, medium length tube solution
N_{Fo} = Fourier number, Equation (8)
N_{Re} = Reynolds number, Γ/μ
N_{Re_c} = critical N_{Re} for wave-free flow
P = function of N_{Fo}, approximate solution, Equation (15)
R = ratio of B absorbed to maximum possible, Equation (6)
S = function of N_{Fo}, short tube solution, Equation (13)
U_s = liquid velocity at gas interface, cm./sec.
x = coordinate, distance perpendicular to gas interface
z = coordinate, distance parallel to gas interface

Greek Letters

Γ = mass flow rate per wetted width, g./(sec.)(cm.)
μ = viscosity, poise
ρ = density, g./cc.
π = 3.14159

Subscripts

B = bulk average concentration at $z = L$
o = initial concentration at $z = 0$
s = surface concentration at $x = 0$

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